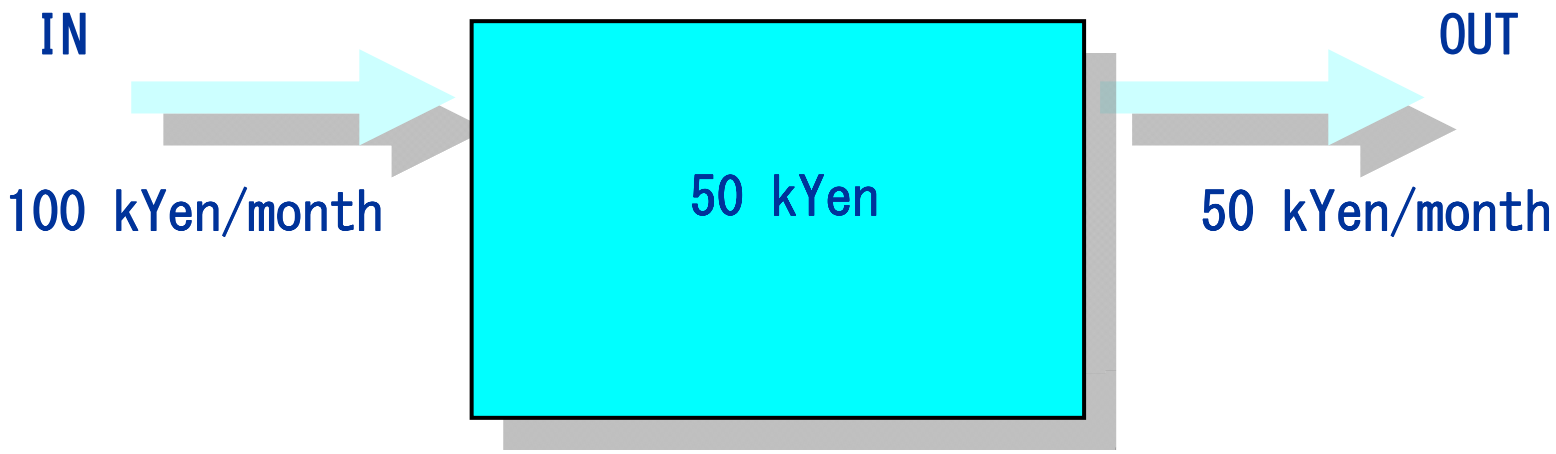
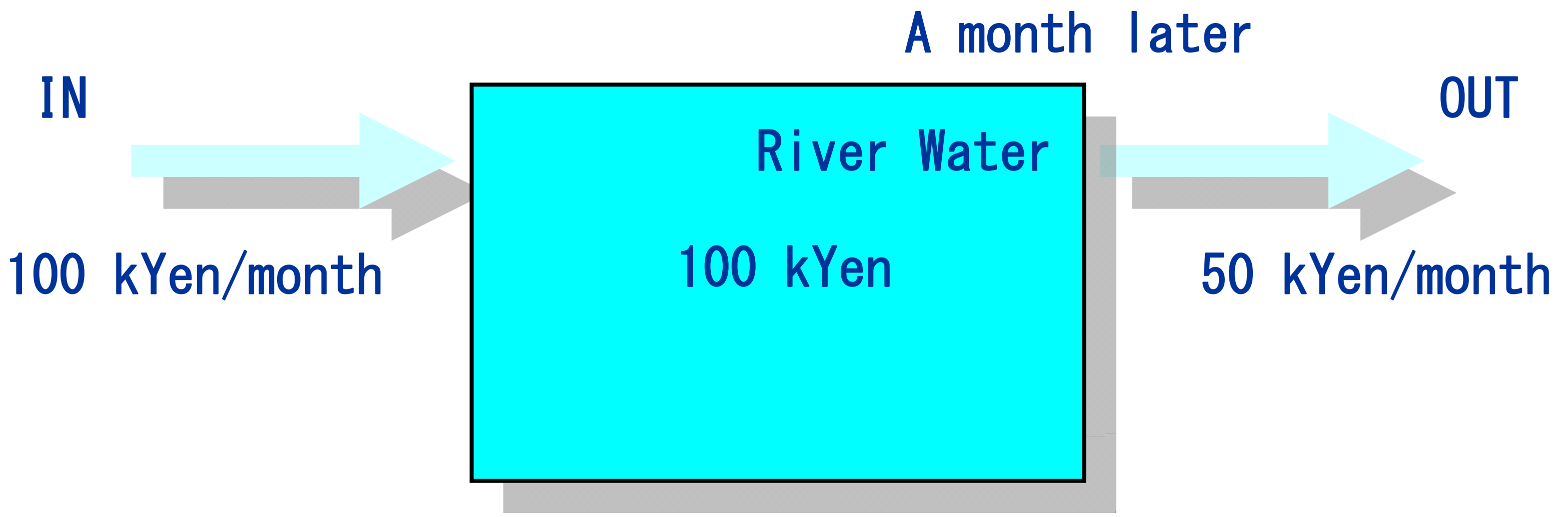
Shinji Ide, the Univ. of Shiga Pref.

INTRODUCTION TO LAKE MODELING

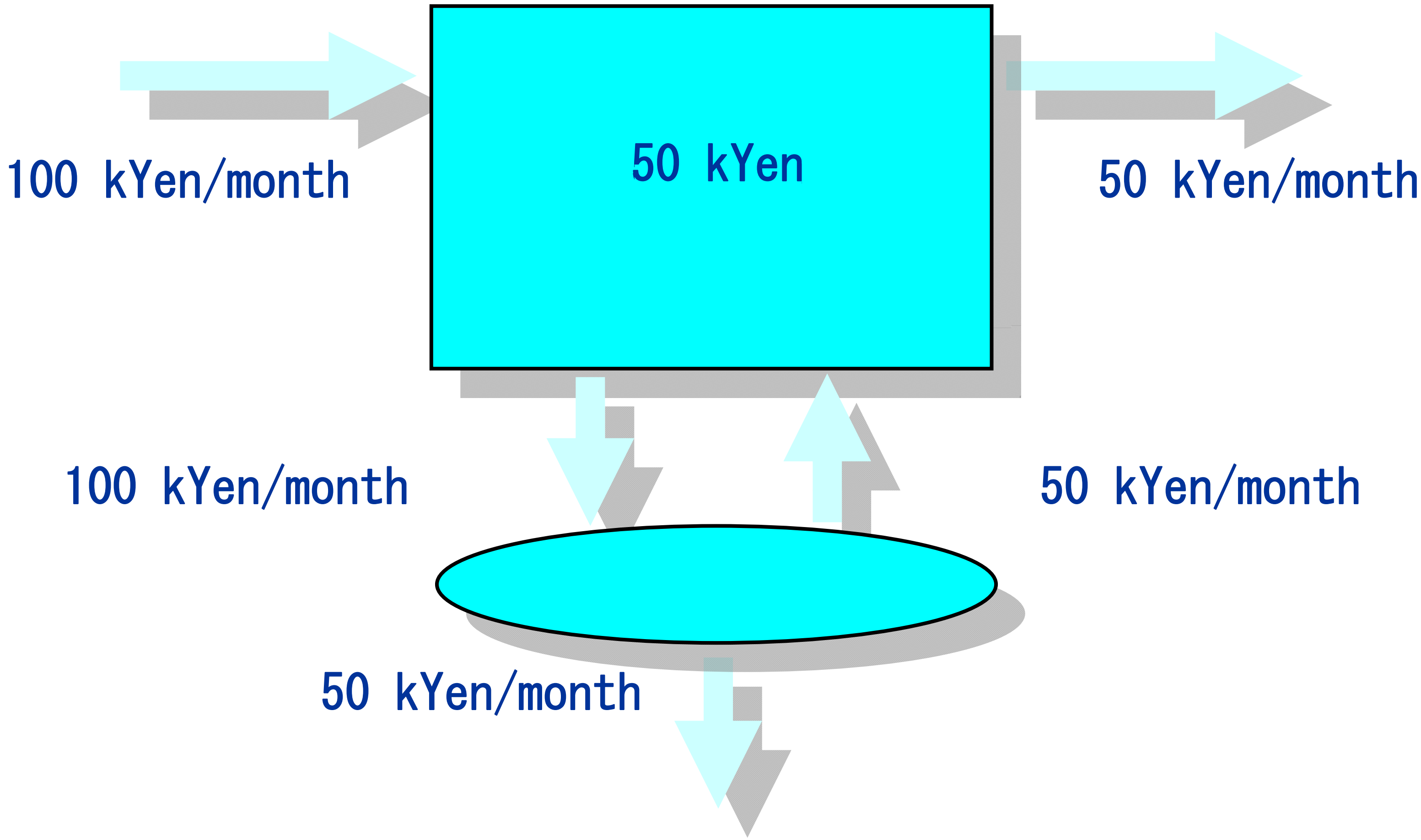
Mass Balance of a river



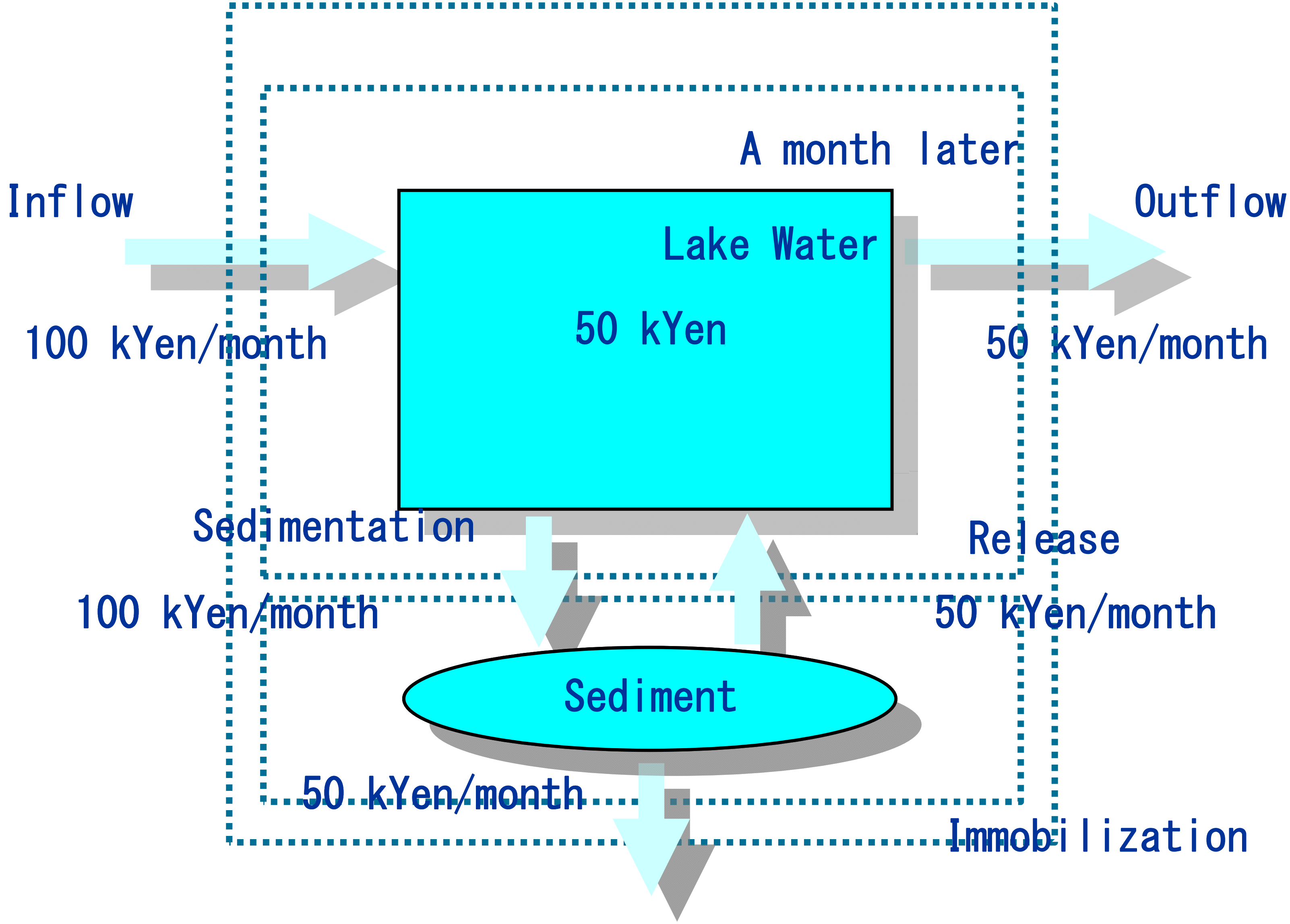
Mass Balance of a river



Mass Balance of a lake

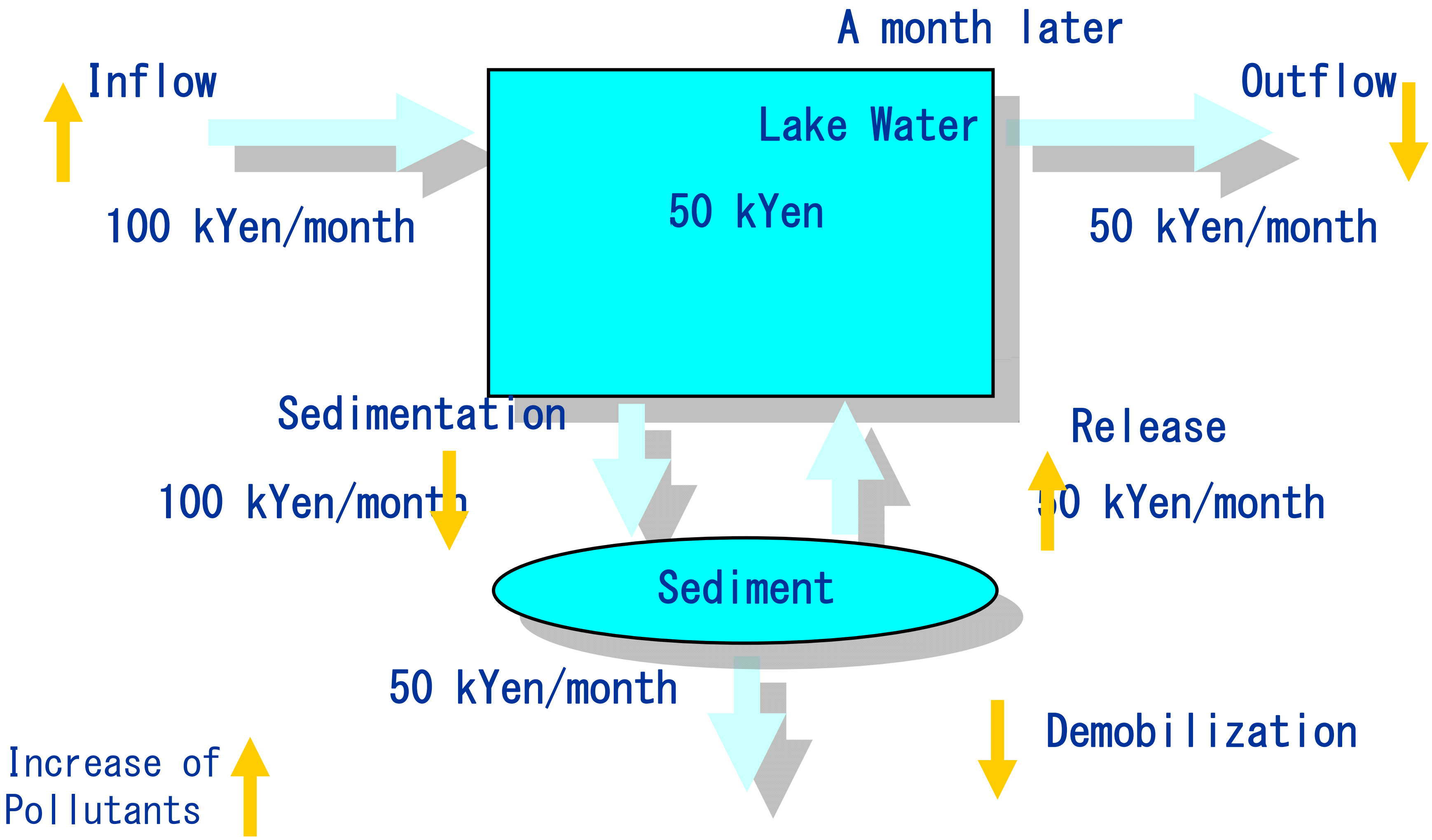


Mass Balance of a lake

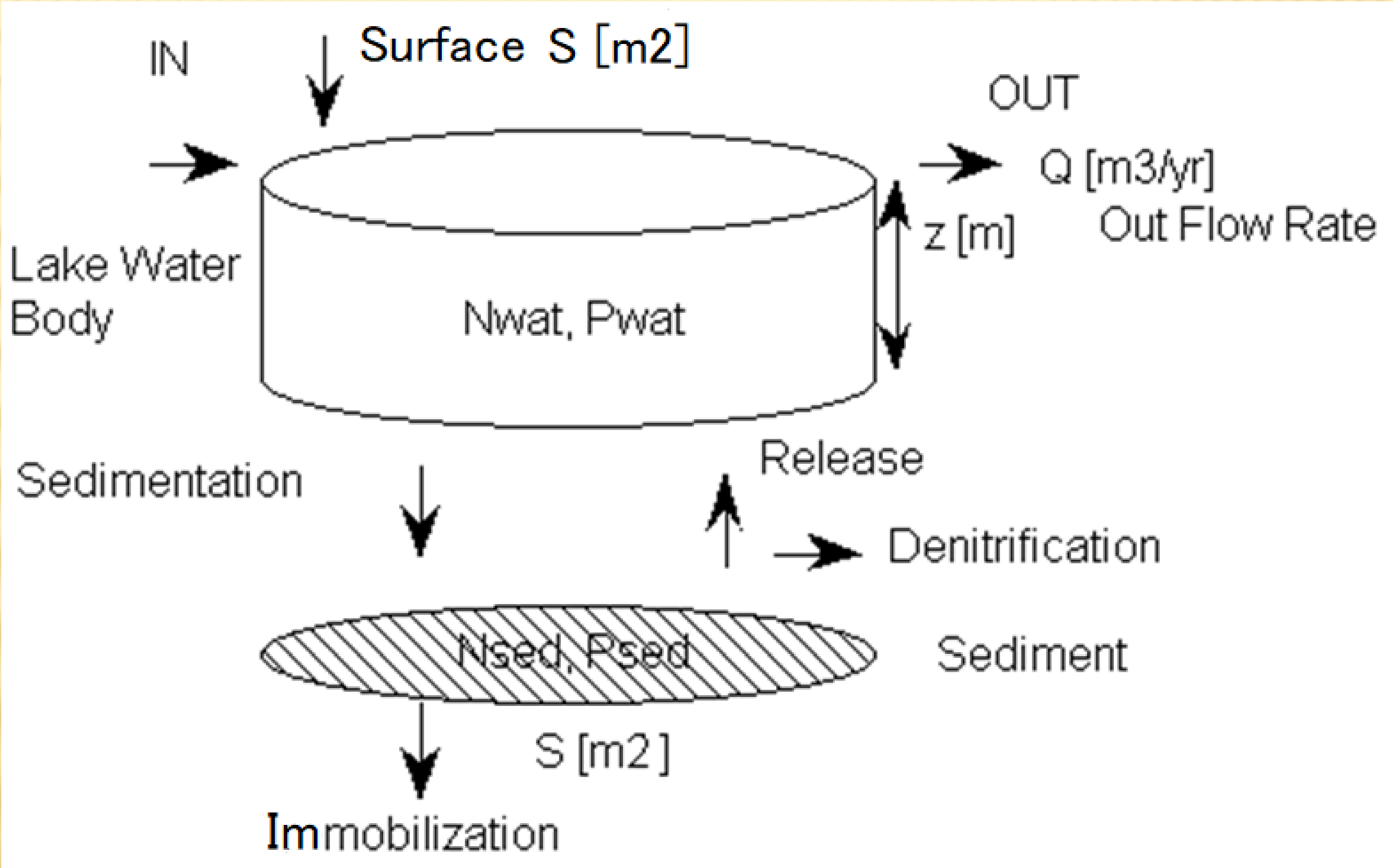


Water Quality Degradation of a lake

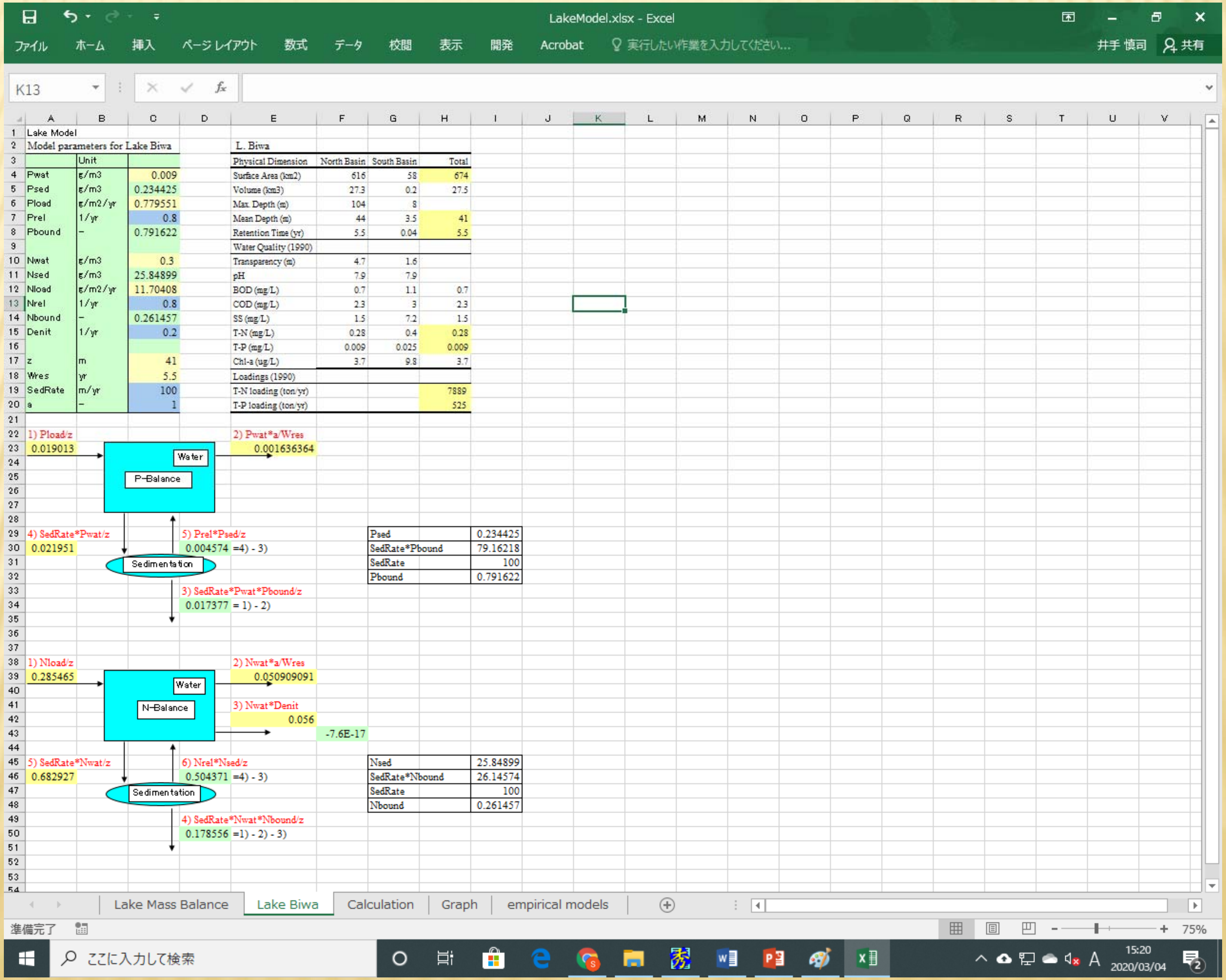
Mass Balance wrt Pollutants



Lake Model


IN
Surface S [m2]
OUT
Q [m3/yr]
Out Flow Rate
z [m]
Lake Water Body
Nwat, Pwat
Sedimentation
Release
Denitrification
Nsed, Psed
Sediment
S [m2]
Immobilization

Lake Model.xlsx



Lake Model

Model parameters for Lake Biwa

	Unit	
Pwat	g/m3	0.009
Psed	g/m3	0.234425
Pload	g/m2/yr	0.779551
Prel	1/yr	0.8
Pbound	–	0.791622
Nwat	g/m3	0.3
Nsed	g/m3	25.84899
Nload	g/m2/yr	11.70408
Nrel	1/yr	0.8
Nbound	–	0.261457
Denit	1/yr	0.2
z	m	41
Wres	yr	5.5
SedRate	m/yr	100
a	–	1

L. Biwa

Physical Dimension	North Basin	South Basin	Total
Surface Area (km2)	616	58	674
Volume (km3)	27.3	0.2	27.5
Max. Depth (m)	104	8	
Mean Depth (m)	44	3.5	41
Retention Time (yr)	5.5	0.04	5.5
Water Quality (1990)			
Transparency (m)	4.7	1.6	
pH	7.9	7.9	
BOD (mg/L)	0.7	1.1	0.7
COD (mg/L)	2.3	3	2.3
SS (mg/L)	1.5	7.2	1.5
T-N (mg/L)	0.28	0.4	0.28
T-P (mg/L)	0.009	0.025	0.009
Chl-a (ug/L)	3.7	9.8	3.7
Loadings (1990)			
T-N loading (ton/yr)			7889
T-P loading (ton/yr)			525

Psed	0.234425
SedRate*Pbound	79.16218
SedRate	100
Pbound	0.791622

Nsed	25.84899
SedRate*Nbound	26.14574
SedRate	100
Nbound	0.261457

Relevant Data of Lake Biwa

Physical Dimension	North Basin	South Basin	Total
Surface Area (km2)	616	58	674
Volume (km3)	27.3	0.2	27.5
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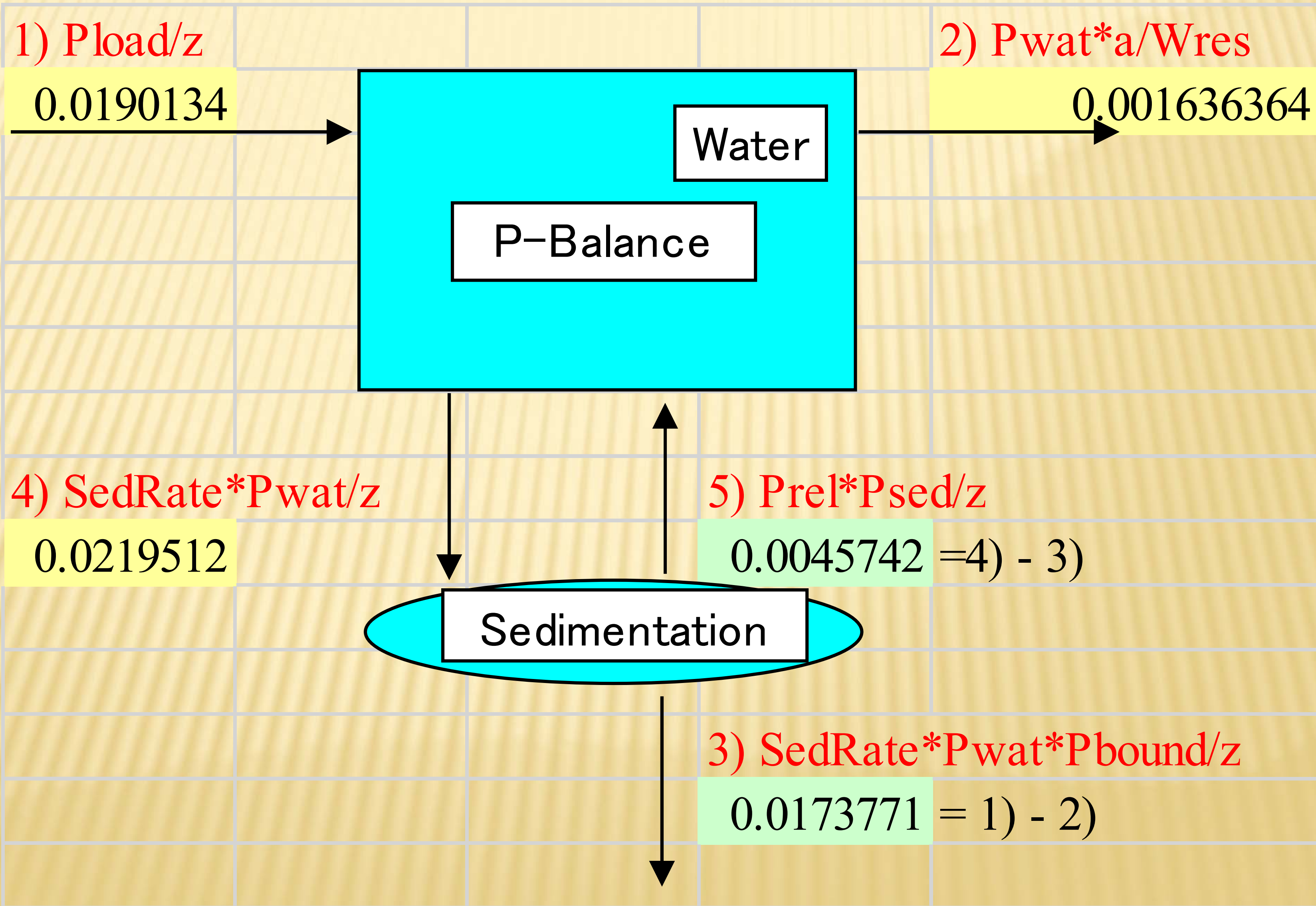
Mass Balacne of Lake Biwa wrt T-P



1) Pload/z

- “Pload” [g/m^2/yr] is phosphorus input “loading” to the lake.
- In limnology, “Pload” is typically defined as [g/m^2/yr]. = (T-P loading)/S (surface area)
- “Pload/z” is hypothetical concentration of T-P coming into the lake. = (T-P loading)/(S*z) = (T-P loading)/V (volume of the lake)
- Where “z” is the mean depth.

1) Pload/z for L. Biwa

- Dividing T-P loading (525 ton/yr) by surface area (674 km^2), "Pload" = 0.7796 [$g/m^2/yr$].
- Dividing "Pload" by "z" (41 m), "Pload/z" = 0.0190 [$g/m^3/yr$].
- The concentration of T-P in the lake water "Pwat" = 0.0090 [g/m^3]

2) Pwat*a/Wres

- Hypothetical concentration of T-P going out of the lake.
- The mass flow of T-P going out of the lake can be calculated with “Pwat*Q” [g/yr], where Q is the outflow rate.
- Dividing “Pwat*Q” by “V” [m^3], “Pwat*Q/V” is to be hypothetical concentration of T-P going out of the lake.

2) Pwat*a/Wres

- "Wres" is the mean residence time of the lake water and defined as "V/Q" in limnology.
- Thus "Pwat*Q/V" = "Pwat/Wres".
- "a" is a correction factor of nutrient output due to thermocline formation.
- In case of strong thermocline, correction with "a" (0-1) would be need for using Pwat as the outflow concentration.

2) Pwat*a/Wres

- If "Wres" is 2 years, it takes 2 years for all water in the lake would be replaced by new water. In other words, within a year, half of the water and contained nutrients in the water would be going out of the lake.
- As a result, hypothetical concentration of T-P going out of the lake would be half of the current concentrations, and can be expressed by "Pwat/Wres".

2) Pwat*a/Wres for L. Biwa

- "Pwat" is 0.0090 [g/m^3], "Wres" is 5.5 [yr], and assuming "a" is 1, then "Pwat*a/Wres" = 0.0016 [g/m^3/yr].
- Recall that "Pload/z" = 0.0190 [g/m^3/yr].
- Only 1/10 of T-P coming into the lake would be going out with the outflow of L. Biwa.

3) SedRate*Pwat*Pbound/z

- Assuming that the lake is in steady state or pseudo-steady state.
- "3) immobilization" can be calculated by 1) – 2): 0.0190 – 0.0016 = 0.0174 [g/m^3/yr].
- Some of nutrients settled down to the sediment would be immobilized there, and never be released back again to the lake water.

4) SedRate*Pwat/z

- "SedRate" is a velocity at which the detritus is (containing T-P) settling down.
- "SedRate/z" is a ratio of detritus to the entire one to be removed from the lake water and moving to the sediment within a year.
- Multiplying "SedRate/z" by "Pwat", "SedRate*Pwat/z" is hypothetical concentration of T-P settling down and reaching the sediment.

3) SedRate*Pwat*Pbound/z

- The ratio of immobilized T-P to the entire T-P settled down to the sediment is defined as "Pbound", which is a dimensionless parameter ranging from 0 to 1.
- Hypothetical concentration of T-P to be immobilized in the sediment would be expressed by 4) "SedRate*Pwat/z"*"Pbound", which is 3) "SedRate*Pwat*Pbound/z".

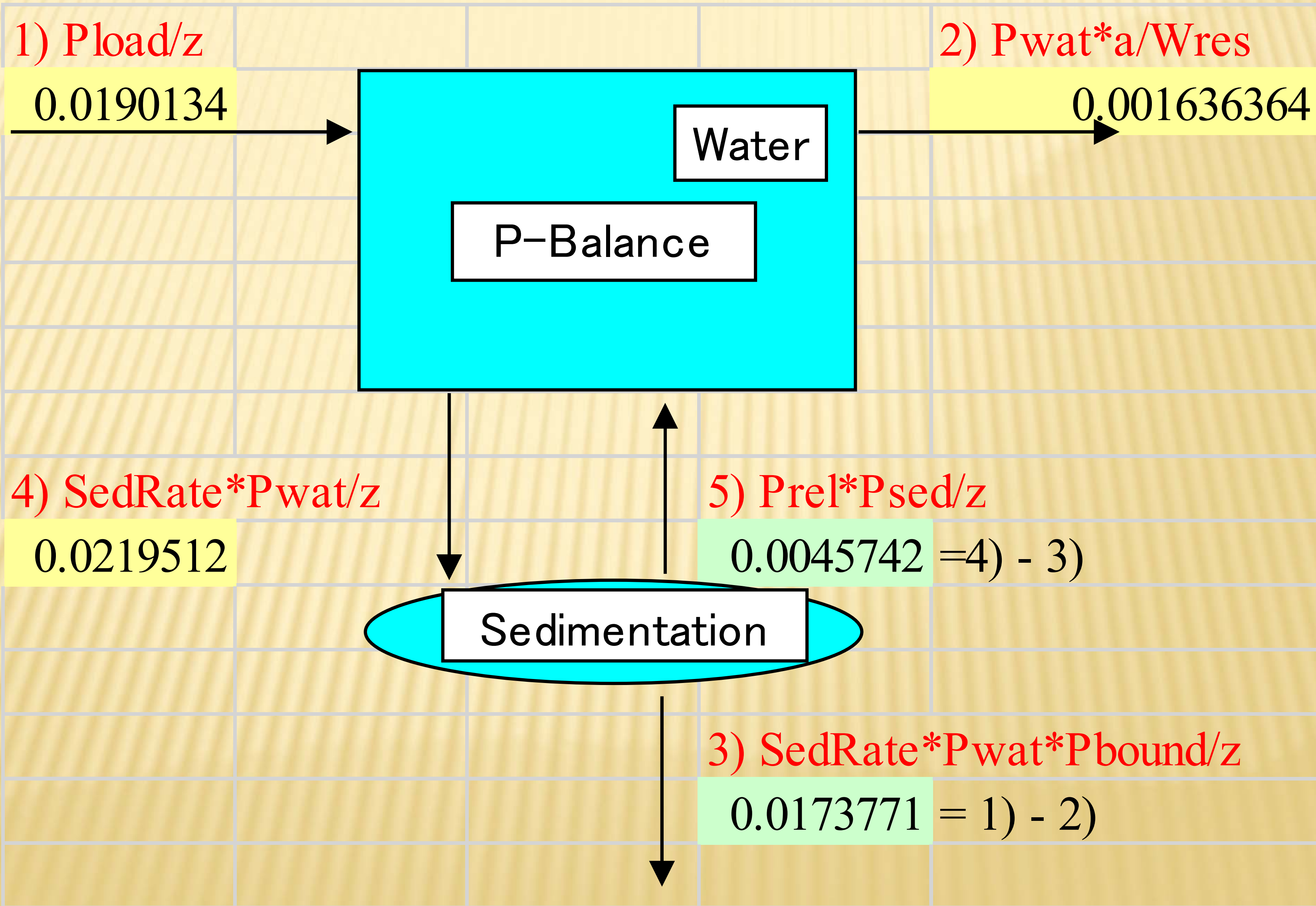
4) SedRate*Pwat/z for L. Biwa

- "SedRate" is 100 [m/yr], "Pwat" is 0.0090 [g/m^3], and "z" is 41 [m].
- "SedRate*Pwat/z" = 100*0.0090/41 = 0.0220 [g/m^3/yr].
- Recall that "Pload/z" = 0.0190 [g/m^3/yr], the value which is very close to "SedRate*Pwat/z" = 0.0220 [g/m^3/yr].

5) Prel*Psed/z

- In steady state, "5) release" can be calculated by 4) – 3): 0.0220 – 0.0174 = 0.0046 [$g/m^3/yr$].
- Some of nutrients being accumulated in the sediment would be <u>released</u> back again to the lake water.
- The release rate of T-P is expressed by a first order reaction: "Psed" (the concentration of T-P in the sediment) [g/m^2]*"Prel" (the sediment release rate) [1/yr].

Mass Balacne of Lake Biwa wrt T-P



Simultaneous Differential Equations

$$\frac{dPwat}{dt} = \frac{Pload}{z} - Pwat \cdot \frac{a}{Wres} - SedRate \cdot Pwat \cdot \frac{1}{z} + Prel \cdot Psed \cdot \frac{1}{z}$$

$$\frac{dPsed}{dt} = SedRate \cdot Pwat \cdot (1 - Pbound) - Prel \cdot Psed$$

$$\frac{dNwat}{dt} = \frac{Nload}{z} - Nwat \cdot \frac{a}{Wres} - Denit \cdot Nwat - SedRate \cdot Nwat \cdot \frac{1}{z} + Nrel \cdot Nsed \cdot \frac{1}{z}$$

$$\frac{dNsed}{dt} = SedRate \cdot Nwat \cdot (1 - Nbound) - Nrel \cdot Nsed$$

Empirical Models

- Chlorophyll [mg/L] = $0.000073 \cdot (Pwat \cdot 1000)^{1.4}$
- Zooplankton [mg/L] = $0.038 \cdot (Pwat \cdot 1000)^{0.64}$
- Fish [mg ww/m^2] = $0.810 \cdot (Pwat \cdot 1000)^{0.71}$
- Average primary production [mg/L/day] = $(10000 \cdot Pwat - 79)/1000$
- Maximum primary production [mg/L/day] = $(20000 \cdot Pwat - 77)/1000$
- Average fish yield [mg ww/m^2/yr] = $7.1 \cdot Pwat$

Assignment (1)

- In Sheet “Lake Mass Balance” of “Lake Model.xlsx:
- (Step 1) Pick up a lake, any lake, in the world for model simulation, and type in the lake name in “D1” yellow cell of the Sheet.
- It is recommended that you choose a lake well-studied.

Assignment (2)

- (Step 2) Find out or identify necessary data for the selected lake, and type in those data in yellow cells of 1) to 7) in “Table: Relevant Data”.
- Those data can be obtained in the literature or on the Internet. Be careful of units!
- If you type in those data, then values of “Table: Model Parameters” will be automatically recalculated.

Assignment (3)

- (Step 3) Confirm that all the values in "Table: Model Parameters" are positive (not negative) and come within appropriate ranges.
- If not, please examine whether "1) Pload/z" > "2) Pwat*a/Wres" and "1) Nload/z" > "2) Nwat*a/Wres", or not. (IN > OUT)
- If not, "6) T-N loading" and/or "7) T-P loading" might be underestimated.

Assignment (4)

- (Step 4) Adjust some parameters, if necessary.
- It is recommended that you should change "8) SedRate" and/or "9) Denit" in the table.
- Confirm the results of two figures of mass balance with respect to T-P and T-N in Sheet "Lake Mass Balance".
- "1) Pload/z" and "4) SedRate*Pwat/z", and "1) Nload/z" and "5) SedRate*Nwat/z" should be close, or at least be in the same magnitude.

Assignment (5)

- In Sheet “Calculation” and “Graph”:
- (Step 5) Confirm that simulation results show the lake is in steady state.
- You copy those values of model parameters and paste them as “values” at “Table: Model parameters” in Sheet “Calculation”. Sheet “Calculation” automatically run the model with the parameters.
- If no problem, all four graphs in Sheet “Graph” should be all flat lines.

Assignment (6)

- (Step 6) Do “What If” simulations.
- For the following A) to G) cases, first predict respectively what happens to “Pwat” & “Psed”, and then examine your prediction with simulation.
- Specifically, are “Pwat” & “Psed” going up or down, or no change? If any change, are those going to reach new stable values or be back to the original values?

Assignment (7)

- A) What If "Pwat" to be zero?
- B) What If "Psed" to be zero?
- C) What If "Pload" to be half?
- D) What If "Pbound" to be half?
- E) What If "z" to be half?
- F) What If "Wres" to be half?
- G) What If "SedRate" to be half?
- Restore changed parameter to the original one before going to the next simulation.