Introduction to Lake Modeling

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This textbook is designed to help readers understand how to develop a simple lake model and run the model for simulation.
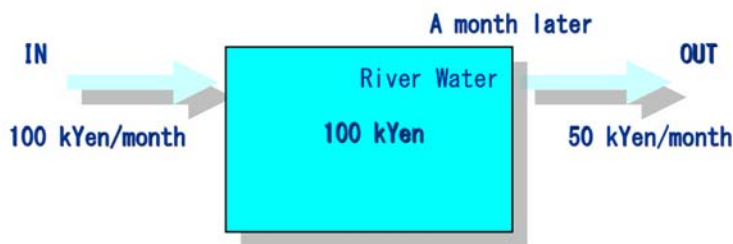
1. Mass Balance

Developing a simple lake model starts from thinking about mass balance of the lake with respect to nutrients.

To understand what mass balance is, let's think of money in your wallet.

As shown in the figure below, let's presume that you have 50 k¥ in the wallet now. If you will gain 100 k¥ and spend 50 k¥ in a month, how much money you would have in the wallet one month later?

The answer is 100 k¥. Money in the wallet is going to be 100 k¥ one month later.

This can be explained with **Mass Balance Equation: (Accumulation = IN - OUT + Reaction).**

In this case, IN is money coming in the wallet, which is 100 k¥, OUT is one going out, which is 50 k¥, and no reaction (no increase or decrease) in the wallet in a month, so that Accumulation would be IN (100) – OUT (50) = 50 k¥ and, as a results, money in the wallet would be 50 + 50 = 100 k¥ one month later.

Likewise, you can think about mass balance of a river with respect to nutrients. The amounts of money in the above example are equivalent to the ones of nutrients either existing in, coming in, or going out of the river. If the river received too much nutrients in compared with the ones going out, then the concentrations of nutrients in the river water would increase, leading to degradation of water

quality.

However, mass balance is much more complicated for lakes. Again, let's think of money in your wallet.

As shown in the figure below, let's presume that you have 50 k¥ in the wallet now. If you will gain 100 k¥ and spend 50 k¥ in a month, how much money you would have in the wallet one month later? But, this time, you have to also assume that you would deposit 100 k¥ in a bank account and withdraw 50 k¥ from the account in a month. In this example, the wallet and bank account are corresponding to water and sediment of the lake, respectively.
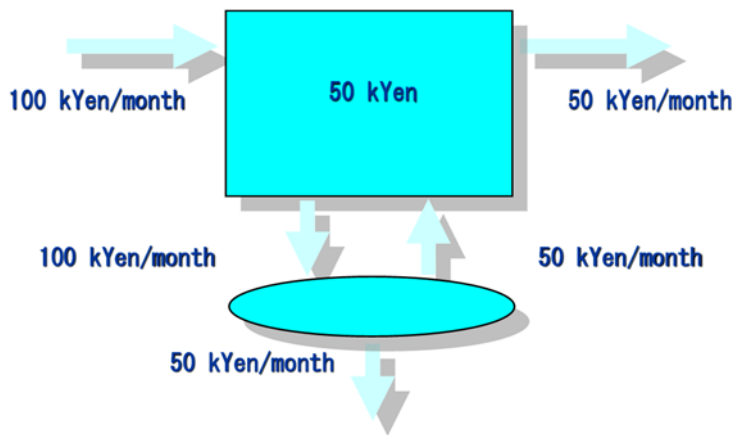
The answer is 50 k¥. Money in the wallet is going to be the same one month later.
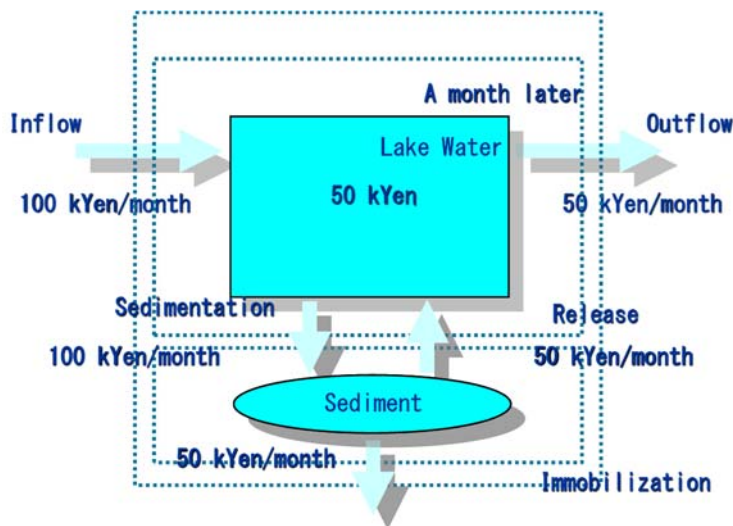
This can also be explained with Mass Balance Equation. But this time, you have to first choose which mass balance you are going to think of. In other words, you need to first identify the boundary of balance.

In the above figure, there are three possible boundaries, as shown in dotted lines. If we drew a boundary around the lake water together with sediment, then, IN is inflow of 100 k¥ and OUT are

outflow of 50 k¥ plus immobilization of 50 k¥. Therefore, Accumulation = IN (100) – OUT (100) = 0, which means no change of money in the wallet one month later.

Likewise, if we drew a boundary around the lake water, then, IN is inflow of 100 k¥ plus release of 50 k¥ and OUT are outflow of 50 k¥ plus sedimentation of 50 k¥. Therefore, Accumulation = IN (100) – OUT (100) = 0. If we drew a boundary around the sediment, then, IN is sedimentation of 100 k¥ and OUT are release of 50 k¥ plus immobilization of 50 k¥. Therefore, Accumulation = IN (100) – OUT (100) = 0.

Although there are three possible boundaries, the accumulation of nutrients in the entire lake system including lake water and sediment, in the lake water, and in the sediment are all the same, zero. In this case, we can say that the lake is in steady state, which means no accumulation or no change in nutrients. As illustrated in these quizzes, the lake system is different from the river system, particularly the former has an additional component, "sediment", and there is big interaction in between the lake water and sediment.

2. Model Description

Lake Model to be used in this text book was originally developed by Prof. Sven E. Jorgensen for the International Lake Environment Committee.
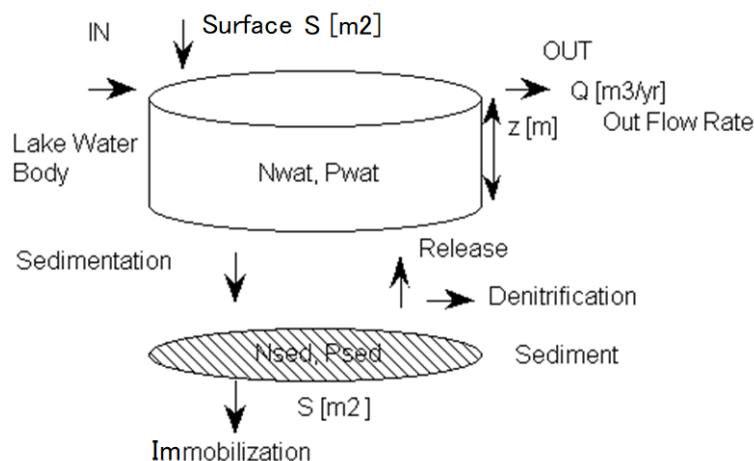

Fig.1: Conceptual diagram for dynamical model of *Lake Model*

Lake Model consists of a combination of two kinds of models: a simple dynamical model, and a set of associated empirical models. The conceptual diagram for the dynamical model is shown in Fig. 1. The dynamical model integrates the pools nitrogen and phosphorus in water and sediment in time as functions of the mass flows. The empirical models are simple regressions made from data of simple physical and chemical characteristics of a number of lakes.

The dynamical model is a modification of the general model made by Vollenweider (1975). While Vollenweider model was only concerned with phosphorus, which is the limiting nutrient in most freshwater bodies, *Lake Model* has included nitrogen as well.

As described in the next chapter, the nitrogen and the phosphorus sub-models are almost identical. The only difference is the denitrification process included in the nitrogen sub-model.

3. Dynamical Model Development

In reading this chapter, please refer to Sheet named "Lake Biwa" in "LakeModel.xlsx", which is an excel file made by the author using *Lake Model* as reference.

Developing the dynamical model, we are going to think of mass balance, again. This time, we are using Lake Biwa in Japan as an example. Physical dimension, water quality, and loadings of L. Biwa are shown in Table 1.

Table 1: Relevant data of L. Biwa

	Symbol	North Basin	South Basin	Total
Physical Dimension				
Surface Area (km^2)	S	616	58	674
Volume (km^3)	V	27.3	0.2	27.5
Max. Depth (m)		104	8	
Mean Depth (m)	z	44	3.5	41
Retention Time (yr)	Wres	5.5	0.04	5.5
Water Quality (1990)				
Transparency (m)		4.7	1.6	
pH		7.9	7.9	
BOD (mg/L)		0.7	1.1	0.7
COD (mg/L)		2.3	3	2.3
SS (mg/L)		1.5	7.2	1.5
Total Nitrogen (mg/L)	Nwat	0.28	0.4	0.28
Total Phosphorus (mg/L)	Pwat	0.009	0.025	0.009
Chl-a (µg/L)		3.7	9.8	3.7
Loadings (1990)				
T-N loading (ton/yr)				7889
T-P loading (ton/yr)				525

Making use of values in Table 1, we are going to estimate mass balance of L. Biwa with respect to total phosphorus (T-P), as shown in Fig. 2.
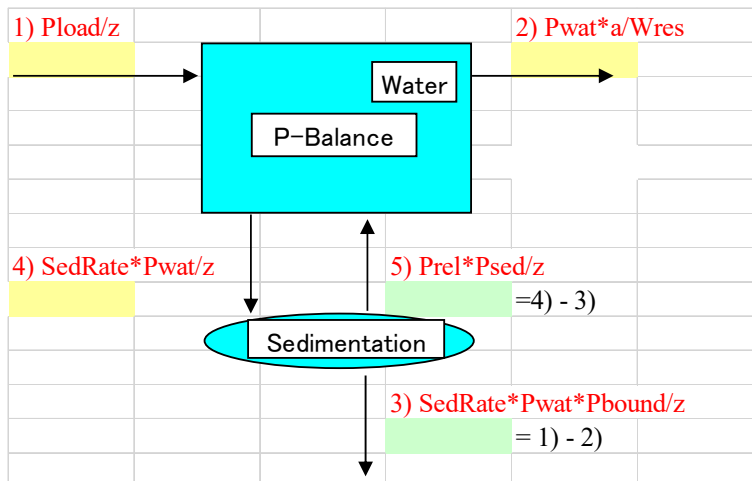
Fig. 2: Mass balance with respect to T-P

Please note that, in the following calculation, we assume that the lake is in steady state or pseudo-steady state, which means the lake is in a stable condition, and thus all the parameters are constant.

1) Pload/z

First, we are going to estimate how much T-P is coming into L. Biwa with "Pload/z". Here "Pload" [$g/m^2/yr$] is phosphorus input "loading" to the lake. In limnology, nutrient loading is typically defined as how many grams of nutrient is coming into the lake per one square meter of lake surface area per year. Dividing total mass of nutrient "T-P loading" coming in by the surface area "S", it makes easier for us to compare loadings among lakes in different size and surface area.

Furthermore, dividing "Pload" = "T-P loading/S" by the mean depth "z", which is the same as dividing "T-P loading" by "S*z" = the volume "V", we can convert the mass of loading to equivalent and hypothetical concentration if the mass coming in would be completely mixed in the lake water. With hypothetical concentrations, it makes easier for us to compare 1) through 5) in Fig. 2.

In case of L. Biwa, "Pload" can be calculated with dividing T-P loading (525 ton/yr) by the surface area (674 km^2), to be 0.7796 [$g/m^2/yr$]. If we divide "Pload" by "z", the mean depth (41 m), then coming up with 1) "Pload/z" = 0.0190 [$g/m^3/yr$].

Recall that the concentration of T-P "Pwat" in the lake water is 0.0090 [$mg/L=g/m^3$] and the unit of [mg/L] is identical with [g/m^3]. T-P is coming into L. Biwa with "Pload/z" = 0.0190 [$g/m^3/yr$] for a year, which means that "Pwat" would be tripled and increase from 0.0090 to 0.0280 [g/m^3] in a year. We can understand that L. Biwa has been receiving that much high loadings of T-P.

2) Pwat*a/Wres

Second, we are going to estimate how much T-P is going out of L. Biwa with "Pwat*a/Wres". Here, "Pwat" is the concentration of T-P in the lake water, "a" is a correction factor of nutrient output due to thermocline formation, and "Wres" is the mean residence time of the lake water.

The mass flow of T-P going out of the lake can be estimated by "Pwat*Q", where Q is the outflow rate from the lake. The unit of "Pwat*Q" is to be [g/m^3]*[m^3/yr] = [g/yr]. Then if one divides "Pwat*Q" by the volume "V" [m^3], then the unit of "Pwat*Q/V" is to be [g/yr]/[m^3] = [$g/m^3/yr$].

Meantime, the mean residence time "Wres" is defined as "V/Q" in limnology. Thus the above "Pwat*Q/V" can be expressed as "Pwat/Wres".

Why "Pwat/Wres" expresses T-P going out of the lake? It can be explained this way. If "Wres" is 2 years for a lake, it takes 2 years for all the water in the lake would be replaced by new water. In other words, within a year, half of the water would be replaced and the same amount of water and nutrients contained in the water would be going out of the lake. As a result, hypothetical concentration of T-P going out of the lake would be half of the present concentrations, and which can be expressed by "Pwat/Wres".

Lastly, "a" is a dimensionless correction factor ranging from 0 to 1. Strong thermocline can be observed in many lakes in summer. With strong thermocline, the concentrations of T-P (or T-N) in outflowing rivers are typically less than the average concentration of T-P in the entire lake. In such case, correction with "a" (0-1) would be need for using Pwat as the outflow concentration. However, for the time being, we assume "a" is 1.

In case of L. Biwa, "Pwat" is 0.0090 [g/m^3], "Wres" is 5.5 [yr], and assuming "a" is 1, then "Pwat*a/Wres" would be 0.0016 [g/m^3/yr]. Recall hypothetical concentration of T-P coming into the lake is 0.0190 [g/m^3/yr]. It means that only one tenth of T-P coming into the lake would be going out with the outflow of L. Biwa.

3) SedRate*Pwat*Pbound/z

If we drew a mass balance boundary around the entire lake system and assume that the lake is in steady state, then immobilization 3) can be calculated by 1) – 2), which is 0.0190 – 0.0016 = 0.0174 [g/m^3/yr]. Here, immobilization means that some of nutrients settled down to the sediment would be captured and immobilized permanently there, and never be released back again to the lake water. We are coming back to the meaning of equation "SedRate*Pwat*Pbound/z", later on.

4) SedRate*Pwat/z

In a lake, large part of T-P (and T-N) are contained in biomass including planktons and fish, and the detritus of the biomass gradually settle down through water column and reach the lake bottom (the sediment). "SedRate" is a velocity how fast the detritus settles down. The hypothetical concentration of such T-P settling down and reaching the sediment can be expressed by "SedRate*Pwat/z".

The reason can be explained this way. If "SedRate" is 50 [m/yr] and "z" is 100 [m], then it takes two years for the entire detritus would be removed and moving to the sediment. In this case, "SedRate/z" would be 1/2 [1/yr], which means half of the entire detritus is to be moving to the sediment within a year. Therefore, multiplying "SedRate/z" by "Pwat" = "SedRate*Pwat/z", we can express hypothetical concentration of T-P to be removed by sedimentation.

Meantime, some part of the detritus settled down and being piled up in the sediment would be decomposed for years, and T-P (and T-N) contained in the detritus would be released back to the lake water again. This process is called "Release". On the other hand, the process for T-P (and T-N) to remain in the sediment is called "Immobilization". The ratio of immobilized T-P to the entire T-P settled down to the sediment is defined as "Pbound", which is a dimensionless parameter ranging from 0 to 1.

In short, hypothetical concentration of T-P to be immobilized in the sediment would be expressed by 4) "SedRate*Pwat/z" times "Pbound", which is 3) "SedRate*Pwat*Pbound/z".

In case of L. Biwa, "SedRate" is 100 [m/yr], "Pwat" is 0.0090 [g/m^3], and "z" is 41 [m], so that "SedRate*Pwat/z" = 100*0.0090/41 = 0.0220 [g/m^3/yr]. Recall that T-P coming into L. Biwa is "Pload/z" = 0.0190 [g/m^3/yr], the value which is very close to "SedRate*Pwat/z" = 0.0220 [g/m^3/yr].
For identifying or estimating "Pbound" and "SedRate" experimentally, please refer to "6. Estimation of Parameters".
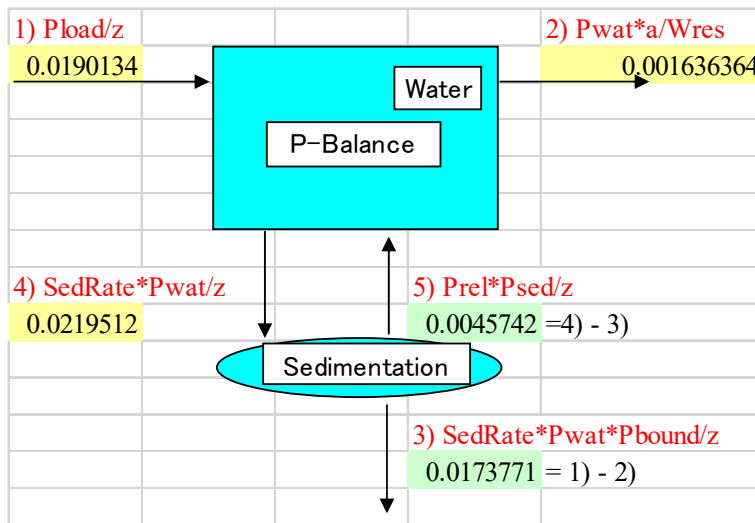
Fig. 3: Mass balance with respect to T-P

5) Prel*Psed/z

If we drew a mass balance boundary around the sediment and assume that the lake is in steady state, then "release 5)" can be calculated by 4) – 3), which is 0.0220 – 0.0174 = 0.0046 [g/m³/yr]. It is assumed that the release rate of T-P from the sediment can be expressed by a first order reaction with respect to the concentration of T-P in the sediment ("Psed") and "Prel" [1/yr], which is the sediment release rate.

In summary, Fig. 3 shows mass balance of L. Biwa with respect to T-P.

In addition, if we add together all items of 1) through 5) then we come up with the following differential equation of "Pwat" with respect to time. The equation is a part of the dynamical model especially for "Pwat". Likewise, we can develop the rest of the dynamical model for "Psed", "Nwat", and "Nsed".

$$\therefore \; {dPwat}/{dt} = {Pload}/{z} - Pwat \cdot {a}/{Wres} - SedRate \cdot Pwat \cdot {1}/{z} + Prel \cdot Psed \cdot {1}/{z} \quad [{g}/{(m^3 \cdot yr)}]$$

4. Dynamical model and empirical models

Dynamical model

More strictly and mathematically, the dynamical model for lakes can be developed in the following manner.

Mass Balance of Lake Water with respect to Phosphorus for Δt [yr]

$$\Delta(Pwat \cdot S \cdot z) = S \cdot z \cdot \Delta Pwat$$

$$= \overbrace{\{Pload \cdot S \cdot \Delta t\}}^{\{IN\}} - \overbrace{\{Pwat \cdot Q \cdot a \cdot \Delta t\}}^{\{OUT\}} - \overbrace{\{SedRate \cdot Pwat \cdot S \cdot \Delta t\}}^{\{Sedimentation\}} + \overbrace{\{Prel \cdot Psed \cdot S \cdot \Delta t\}}^{\{Release\}} \quad [g]$$

where:

Pwat [mg/L]   total phosphorus in the water;
S [m²]   the surface area of the lake;
z [m]   the mean depth of the lake;
Pload [g/m²/yr]   phosphorus input to the lake;
Δt [yr]   a time segment;
Q [m³/yr]   the outflow rate;
a [-]   a correction factor of nutrient output due to thermocline formation;
SedRate [m/yr]   the mean sedimentation rate;
Prel [1/yr]   the sediment release rate of phosphorus; and
Psed [g/m²]   phosphorus in the sediment.

By multiplying both side of the above equation with $\frac{1}{(\Delta t \cdot S \cdot z)}$

$$\frac{\Delta Pwat}{\Delta t} = \frac{Pload}{z} - Pwat \cdot \frac{a}{Wres} - SedRate \cdot Pwat \cdot \frac{1}{z} + Prel \cdot Psed \cdot \frac{1}{z} \quad [\frac{g}{(m^3 \cdot yr)}]$$

where

$$\frac{Q}{(S \cdot z)} = \frac{Q}{V} = \frac{1}{Wres}$$

V [km³]   volume of the lake
Wres [yr]   the mean residence time of the water.

$$\lim_{\Delta t \to 0} \frac{\Delta Nwat}{\Delta t} = \frac{dNwat}{dt}$$

$$\therefore \frac{dPwat}{dt} = \frac{Pload}{z} - Pwat \cdot \frac{a}{Wres} - SedRate \cdot Pwat \cdot \frac{1}{z} + Prel \cdot Psed \cdot \frac{1}{z} \quad [\frac{g}{(m^3 \cdot yr)}]$$

Mass Balance of Lake Sediment with respect to Phosphorus for Δt [yr]

$$\Delta(Psed \cdot S) = S \cdot \Delta Psed$$

$$= \overbrace{\{SedRate \cdot Pwat \cdot S \cdot \Delta t\}}^{\{IN\}} - \overbrace{\{Prel \cdot Psed \cdot S \cdot \Delta t\}}^{\{OUT\}} + \overbrace{\{Pbound \cdot SedRate \cdot Pwat \cdot S \cdot \Delta t\}}^{\{immobilization\}} \quad [g]$$

where:
Pbound [-]   the ratio of immobilized phosphorus in the sediment.

By multiplying both side of the above equation with $\frac{1}{(\Delta t \cdot S)}$ and $\lim_{\Delta t \to 0} \frac{\Delta Psed}{\Delta t} = \frac{dPsed}{dt}$

$$\therefore \frac{dPsed}{dt} = SedRate \cdot Pwat \cdot (1 - Pbound) - Prel \cdot Psed \quad [\frac{g}{(m^2 \cdot yr)}]$$

Mass Balance of Lake Water with respect to Nitrogen for Δt [yr]

$$\Delta(Nwat \cdot S \cdot z) = S \cdot z \cdot \Delta Nwat$$

$$= \overbrace{\{Nload \cdot S \cdot \Delta t\}}^{\{IN\}} - \overbrace{\{Nwat \cdot Q \cdot a \cdot \Delta t\}}^{\{OUT\}} - \overbrace{\{Denit \cdot Nwat \cdot S \cdot z \cdot \Delta t\}}^{\{Denitrification\}} - \overbrace{\{SedRate \cdot Nwat \cdot S \cdot \Delta t\}}^{\{Sedimentation\}} +$$

$$\overbrace{\{Nrel \cdot Nsed \cdot S \cdot \Delta t\}}^{\{Release\}} \quad [g]$$

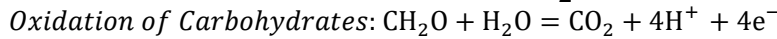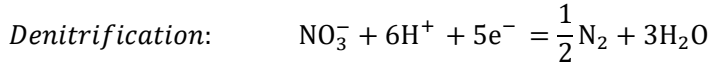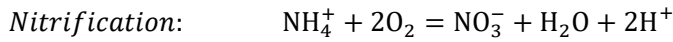Note that a first order reaction is assumed for denitrification.

where:

Nwat [mg/L]   total nitrogen in the water;
Nload [g/m²/yr]   nitrogen input to the lake;
Denit [1/yr]   the reaction coefficient of denitrification);
Nrel [1/yr]   the sediment release rate of nitrogen; and
Nsed [g/m²]   nitrogen in the sediment.

By multiplying both side of the above equation with $\frac{1}{(\Delta t \cdot S \cdot z)}$ and $\lim\limits_{\Delta t \to 0} \frac{\Delta Nwat}{\Delta t} = \frac{dNwat}{dt}$

$\therefore \frac{dNwat}{dt} = \frac{Nload}{z} - Nwat \cdot \frac{a}{Wres} - Nwat \cdot Denit - SedRate \cdot Nwat \cdot \frac{1}{z}$

$+ Nrel \cdot Nsed \cdot \frac{1}{z} \quad [\frac{g}{(m^3 \cdot yr)}]$

The denitrification is a series of biological reactions as shown below. The reaction rate is expressed by a first order reaction with respect to "T-N".

$Nitrification:$ $NH_4^+ + 2O_2 = NO_3^- + H_2O + 2H^+$

$Denitrification:$ $NO_3^- + 6H^+ + 5e^- = \frac{1}{2}N_2 + 3H_2O$

$Oxidation\ of\ Carbohydrates:$ $CH_2O + H_2O = CO_2 + 4H^+ + 4e^-$

Mass Balance of Lake Sediment with respect to Nitrogen for Δt [yr]

$\Delta(Nsed \cdot S) = S \cdot \Delta Nsed$

$= \overbrace{\{SedRate \cdot Nwat \cdot S \cdot \Delta t\}}^{\{IN\}} - \overbrace{\{Nrel \cdot Nsed \cdot S \cdot \Delta t\}}^{\{OUT\}} + \overbrace{\{Nbound \cdot SedRate \cdot Nwat \cdot S \cdot \Delta t\}}^{\{immobilization\}} \quad [g]$

where:
Nbound [-]   the ratio of immobilized nitrogen in the sediment.

By multiplying both side of the above equation with $\frac{1}{(\Delta t \cdot S)}$ and $\lim\limits_{\Delta t \to 0} \frac{\Delta Nsed}{\Delta t} = \frac{dNsed}{dt}$

$\therefore \frac{dNsed}{dt} = SedRate \cdot Nwat \cdot (1 - Nbound) - Nrel \cdot Nsed \quad [\frac{g}{(m^2 \cdot yr)}]$

Empirical Models

The empirical models are a number of relations made from statistical regression analyses (Edmondson, 1986).

Chlorophyll [mg/L] $= 0.000073 \cdot (Pwat \cdot 1000)^{1.4}$
Zooplankton [mg/L] $= 0.038 \cdot (Pwat \cdot 1000)^{0.64}$
Fish [mg ww/m²] $= 0.810 \cdot (Pwat \cdot 1000)^{0.71}$
Average primary production [mg/L/day]= (10000·Pwat - 79)/1000
Maximum primary production [mg/L/day]   = (20000 ·Pwat - 77)/1000
Average fish yield [mg ww/m²/yr]   = 7.1·Pwat

Lake Model is supplied with an algorithm to decide if phosphorus and/or nitrogen are limiting nutrients to the phytoplankton growth. The algorithm is based on the knowledge about the mean internal cell ratios of nutrients in phytoplankton. The algorithm is based on the following rules:

IF total N >= 10·total P then P is the limiting nutrient

IF total N <= 5·total P then N is the limiting nutrient

IF 5·total P < total N < 10·total P then P is the limiting nutrient

5. Parameter Tuning (How to use LakeModel.xlsx)

The dynamical model consists of 4 differential equations:

$$dPwat/_{dt} = Pload/_z - Pwat \cdot a/_{Wres} - SedRate \cdot Pwat \cdot 1/_z + Prel \cdot Psed \cdot 1/_z \ [g/_{(m^3 \cdot yr)}]$$

$$dPsed/_{dt} = SedRate \cdot Pwat \cdot (1 - Pbound) - Prel \cdot Psed \ [g/_{(m^2 \cdot yr)}]$$

$$dNwat/_{dt} = Nload/_z - Nwat \cdot a/_{Wres} - Nwat \cdot Denit - SedRate \cdot Nwat \cdot 1/_z$$
$$+ Nrel \cdot Nsed \cdot 1/_z \ [g/_{(m^3 \cdot yr)}]$$

$$dNsed/_{dt} = SedRate \cdot Nwat \cdot (1 - Nbound) - Nrel \cdot Nsed \ [g/_{(m^2 \cdot yr)}]$$

Although we need to solve the above simultaneous differential equations with numerical analysis for running the dynamical model, we are not going into the details of numerical analysis in this textbook.

On the other hand, we must specify parameters for running the model. Hereinafter, using Sheet "Lake Mass Balance" in "LakeModel.xlsx", how to identify or estimate the parameters will be explained.

With Sheet "heet be s Balanceexp"Table: Relevant data"aand "Table: Model Parameters" (Table 2 and 3) show relevant data and model parameters for Lake Suwa as an example. As for a selected lake, being careful of units, find out and change those data of 1) to 7) in "Table: Relevant data"abTable 2). Then, parameters for Lake Model in "Table: Model Parameters" (Table 3) will be automatically calculated. If you get negative numbers in any cells in "Table: Model Parameters" (Table 3), you may change the values of 8) and/or 9) in "Table: Model Parameters" (Table 3).

Table 2: Relevant data of L. Suwa

Physical Dimension	
1) Surface Area (km2)	13.3
Volume (km3)	0.063
Max. Depth (m)	7.2
2) Mean Depth (m)	4.7
3) Retention Time (yr)	0.11
Water Quality (1983-85)	
Transparency (m)	1.1
pH	8.4

BOD (mg/L)	-
COD (mg/L)	6.1
SS (mg/L)	12.8
4) T-N (mg/L)	1.3
5) T-P (mg/L)	0.094
Chl-a (ug/L)	65.2
Loadings (1984)	
6) T-N loading (ton/yr)	832
7) T-P loading (ton/yr)	111

Table 3: Model parameters for L. Suwa

	Unit				Nload	g/m^2/yr	62.556391	Given
Pwat	g/m^3	0.094	Given		Nrel	1/yr	0.8	Assumed
Psed	g/m^2	6.3381237	Estimated		Nbound	- (0-1)	0.6065534	Estimated
Pload	g/m^2/yr	8.3458647	Given		9) Denit	1/yr	0.2	Assumed
Prel	1/yr	0.8	Assumed					
Pbound	- (0-1)	0.4605852	Estimated		Z	M	4.7	Given
					Wres	yr	0.11	Given
Nwat	g/m^3	1.3	Given		8) SedRate	m/yr	100	Assumed
Nsed	g/m^2	155.623	Estimated		a	- (0-1)	1	Assumed

Once, we get all positive numbers in "Table: Model Parameters" (Table 3), then copy those numbers and paste them as "values" at "Table of Model parameters" in Sheet "Calculation". Be careful, choose "values" in the paste option.

Sheet "Calculation" automatically run the model with those parameters. You can take a look at simulation results in Sheet "Graph". The sheet shows simulation results of Pwat, Psed, Nwat, and Nsed for 20 years. Please note that in this time, the graphs in Sheet "Graph" should be all flat lines, as shown in Fig. 4, since parameters in "Table: Model Parameters" (Table 3), were estimated with assumption of steady state, which means no change in concentrations of T-P and T-N in both lake water and sediment.
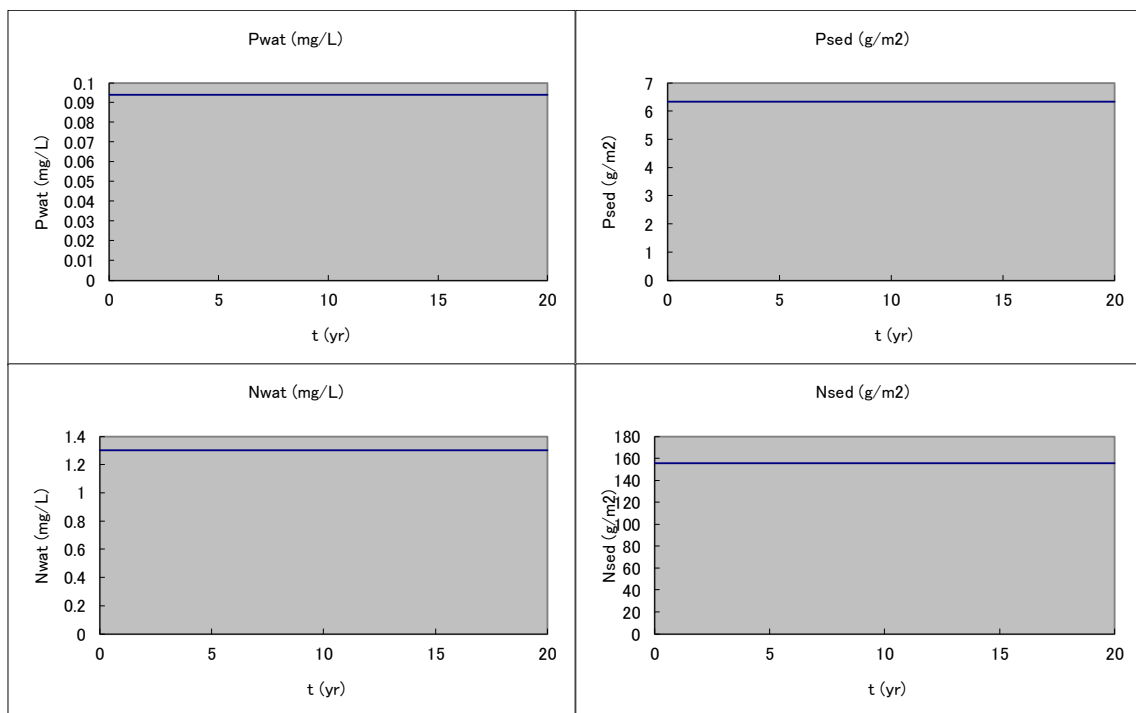

Fig. 4 Simulation results of Pwat, Psed, Nwat, and Nsed in steady state.

Once, you got all flat-line graphs in Sheet "Graph", which means that you have finished parameter tuning and you are ready for simulations. Doing "What If" simulations, try to change values of some parameters in "Table: Model parameters" in Sheet "Calculation" and see the simulation results in Sheet "Graph".

As matter of course, you may change values of 1) to 7) in "Table: Relevant data" (Table 2) for another lake, and follow the above-mentioned procedure, with which you can run the model for

another lake.

6. Estimation of Parameters

For your reference, there are several ways to identify or estimate "Nbound" and "Pbound" and "SedRater" experimentally, as illustrated below.

Nbound and Pbound

Fig. 5 shows the typical concentration profiles of nitrogen or phosphorus in the sediment. The concentration of T-P or T-N is maximum, "x", at the top layer of the sediment and going down along the depth of the sediment, finally reaching specific values of "y". The difference between the "x" and "y" represents a portion of nutrients, which have been released back to the lake water. The values of "Nbound" or "Pbound" in *Lake Model* can be defined by the ratio of "y" to "x" in the figure.
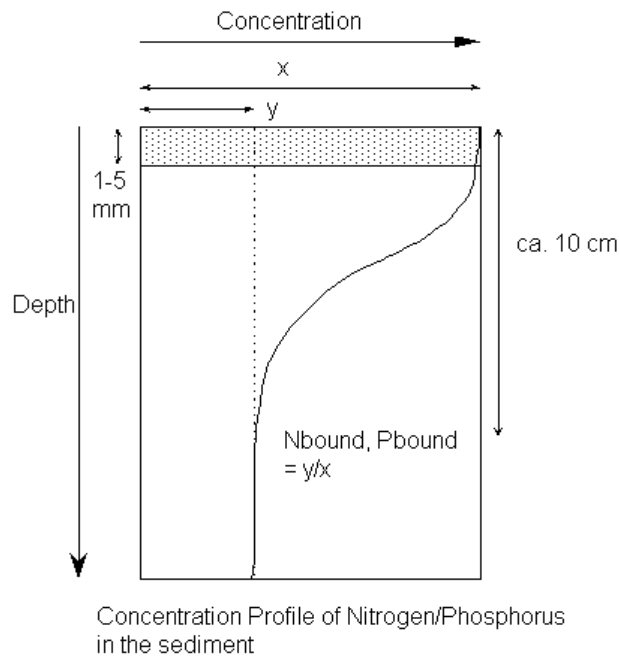

Fig. 5: Concentration profiles of nitrogen and phosphorus in the sediment

SedRate

Should you wish to estimate "SedRate", put a tray of surface area of s [m^2] into the lake for a certain period, Δt [yr]. During the period, some amount of detritus will be piled up on the tray. Let assume that the T-P of this detritus is Pt [g]. Then Pt can be expressed by the following equation with "Pwat" and "SedRate", based on the assumptions of *Lake Model*:

$$Pt = SedRate \cdot Pwat \cdot s \cdot \Delta t$$

Consequently, "SedRate" can be calculated by the following equation:

$$SedRate = \frac{Pt}{Pwat \cdot s \cdot \Delta t}$$

"SedRate" can be also crosschecked by the following equation:

$$\text{SedRate} = \frac{\text{Nt}}{\text{Nwat} \cdot \text{s} \cdot \Delta \text{t}}$$

where Nt [g] is T-N of the detritus.

7. Assignment

Using "LakeModel.xlsx", identify or estimate model parameters of a lake, and answer to the following quizzes with running the model. Please note that relevant data of Lake Suwa (Japan) were provided in Sheet "Lake Mass Balance" at the beginning. You will need to change those data for your lake.

In Sheet "Lake Mass Balance":

(Step 1) Pick up a lake, any lake, in the world for model simulation, and type in the lake name in "D1" yellow cell of the Sheet.

Please note that you will need those data listed 1) to 7) in Table 2 to run the model. You are recommended that you choose a lake well-studied.

(Step 2) Find out or identify necessary data for the selected lake, and type in those data in yellow cells of 1) to 7) in "Table: Relevant Data".

Particularly, you must specify and type in 1) Surface Area, 2) Mean Depth, 3) Retention Time, 4) T-N, 5) T-P, 6) T-N loading, 7) T-P loading, which data can be obtained in the literature or on the Internet.

Be careful of units! You may need to convert values to align the units listed in the table. If you type in those data, then values of "Table: Model Parameters" will be automatically recalculated.

(Step 3) Confirm that all the values in "Table: Model Parameters" are positive (not negative) and come within appropriate ranges.

If not, please first to examine whether the value of "1) Pload/z" is greater than one of "2) Pwat*a/Wres" and the value of "1) Nload/z" is greater than one of "2) Nwat*a/Wres", or not. As matter of course, IN must be greater than OUT. If OUT is greater than N, then the concentrations of T-P and/or T-N keep decreasing. Otherwise, you should suspect the values of "6) T-N loading" and/or "7) T-P loading", which might be underestimated. If those values are suspected to be underestimated and no additional evidence to correct the values, then it is recommended you should change the lake to another, and go back again to Step (1).

(Step 4) Adjust some parameters, if necessary.

If the above condition is satisfied but still some values are negative or out of appropriate range, then it is recommended that you should change the values of "8) SedRate" and/or "9) Denit" in the table. Those are kind of literature values, and particularly the default value of "8) SedRate" is the one estimated for L. Suwa. "8) SedRate" may differ from one lake to another.

In Sheet "Lake Mass Balance", there are two figures of mass balance with respect to T-P and T-N below the tables. With those figures, you can examine by yourself how mass balance of the lake would change when changing value of "8) SedRate".

Here is a good rule of thumb: the values of "1) Pload/z" and "4) SedRate*Pwat/z", and the ones of "1) Nload/z" and "5) SedRate*Nwat/z" should be close, or at least be in the same magnitude. You should adjust the value of "8) SedRate" or other parameters to meet this condition.

Why is the above rule rational? Because it can be said that most of nutrients coming into the lake will be first settling down to the sediment, and then nutrients released back from the sediment to the lake water will be going out of the lake.

You may also be able to change "Prel", "Nrel", and "a" in "Table: Model Parameters" although it is rare case.

In Sheet "Calculation" and "Graph":

(Step 5) Confirm that simulation results show the lake in steady state.

Once, we get all appropriate values in the table, then you copy those values and paste them as "values" at "Table: Model parameters" in Sheet "Calculation". Be careful, choose "values" in the paste option.

Sheet "Calculation" automatically run the model with the parameters. You can check the simulation results in Sheet "Graph". If no problem, all four graphs in the sheet should be flat lines, as shown in Fig. 4.

(Step 6) Do "What If" simulations.

For the following seven cases of A) to G), predict first respectively what happens to "Pwat" and "Psed", and then examine your prediction by changing parameters in "Table: Model parameters" in Sheet "Calculation" and see the simulation results. Specifically, predict if the concentrations of "Pwat" and/or "Psed" are going up or down, or no change? If any change, are those concentrations going to reach new stable values or be back to the original values?

A) What If "Pwat" to be zero?

B) What If "Psed" to be zero?　(To see the effects of dredging.)

C) What If "Pload" to be half?　(To see the effects of sewage system.)

D) What If "Pbound" to be half?

E) What If "z" to be half?

F) What If "Wres" to be half?

G) What If "SedRate" to be half?

Please note that you change one parameter in each case, and restore it to the original one before going to the next simulation.